Background

This document guides districts as they support students in becoming powerful mathematical thinkers, and in experiencing success on the SAT Suite of Assessments. The SAT subgroup of the Michigan Association of Intermediate School Administrators’ Mathematics Leadership Team, a group of ISD mathematics education consultants from across the state, collaboratively developed this resource. The document is part of a larger list of curated SAT math resources. They can be accessed online by visiting this URL: http://bit.ly/2ovDELW.

“The most important preparation for the redesigned SAT will occur in the course of classroom activities,” notes The Redesigned SAT: Teacher Implementation Guide (p. 72). These recommendations, then, do not only support students’ understanding of the content and practices necessary for success on the SAT assessments; they also embody effective mathematics instructional practices. These instructional practices draw from research documents describing the nature of high-quality mathematics instruction, including NCTM’s 2014 Principles to Actions; the IES Practice Guide Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students; and Michigan’s mathematics content and practice standards. It is important to note: Preparation for the SAT must take place as part of a comprehensive plan to deliver high-quality mathematics instruction for all students.

The recommendations in this document should be implemented as part of daily, high-quality mathematics experiences for students. They do not require any specific curriculum resources. Also, these recommendations should be integrated across multiple disciplines (e.g., graph interpretation in social studies courses, or algebraic formulas in physics).

Effective preparation for high-stakes assessments happens neither overnight nor in a specific course, delivered just before the assessment. For students to be successful in mathematics and on assessments like the SAT, skills and habits of mind must develop over time (McTighe & Wiggins, 2013). Students should have experiences consistent with these recommendations through multiple grade levels and courses, thus ensuring coherent pathways to preparedness for the SAT.
The nature of mathematical tasks can have a large impact on students’ learning (Stein, Smith, Henningsen, & Silver, 2009). Whether in “real world” contexts or not, cognitively demanding tasks are those that have no single defined solution path or procedure. They require students to discuss and reason about mathematical content, structure, and processes. As students engage with these tasks, they develop habits of mind like those called for in the Standards for Mathematical Practice. (Individual SMPs are recommended parenthetically throughout this document, next to recommendations.) The focus should allow students to make sense of these problems on their own and in collaboration with others. By focusing learning on problem-solving, students can develop deep, flexible knowledge that can be applied to novel situations. This kind of knowledge is highly useful in mathematics and on assessments such as the SAT (McTighe & Wiggins, 2013).

**Teachers should:**
- Incorporate cognitively demanding tasks into daily instruction and assessments
- Use high-demand tasks to build the conceptual understanding of important mathematical content
- Allow students to engage with high-demand tasks individually and collaboratively
- Foster productive discussion (groups and peer to peer) around solution methods (SMP 3)
- Encourage non-algebraic solution methods to problems before resorting to algebraic manipulations (SMP 2, SMP 7)
- Modify curricula to make them more cognitively demanding
- Model close and critical reading, and provide opportunities for students to practice with feedback
- Support sense-making and perseverance in problem solving (SMP 1), through effective questioning strategies (Herbel-Eisenmann & Breyfogle, 2005)
- Plan lessons (Smith, Bill, & Hughes, 2008) to support students’ sense-making.

### Key Consideration

*By focusing learning on problem-solving, students can develop deep, flexible knowledge that can be applied to novel situations.*
Practice 2: Incorporate and connect multiple mathematical representations into curriculum, instruction, and assessment.

Few strategies to promote access and enduring understanding are more powerful than the incorporation of multiple representations (National Council of Teachers of Mathematics, 2000). The effective use of graphical, pictorial, algebraic, tabular, and verbal representations is vital to support students in making sense of mathematics.

Discussing the connections between and usefulness of these representations allows students to build the habits of mind articulated in the SMPs. This increases their flexibility and efficiency in problem solving. In particular, technology and other tools (SMP 5) help students generate and translate meaning among representations. This practice is intimately related to Recommendation 1, in that multiple representations are a vital part of engaging with and making sense of cognitively demanding tasks.

Teachers should:

- Implement cognitively demanding tasks that require students to make use of multiple representations (e.g., tables, graphs, equations, words, and pictures) (SMP 4)
- Ask questions that allow students to go past the production of representations, toward making connections between them and analyzing the usefulness of each (SMP 2, SMP 7)
- Ensure students’ access to a variety of tools and technology, to create and analyze multiple representations (SMP 5, SMP 6)
- Develop classroom norms of collaboration, focusing on multiple representations in the problem-solving process (Horn, 2012)
- Carefully plan for and encourage the use of multiple representations as investigatory strategies
- Ensure clarity in the curriculum about which types of representations (e.g., models, graphs, pictures, tools, etc.) all students will engage with, when working on important content
Practice 3: Focus students’ attention on mathematical structure.

Understanding mathematical structure—e.g., features of mathematical objects such as the symmetry of graphs of quadratic functions—can greatly improve students’ efficiency in developing solution methods (SMP 2, SMP 7). Taking advantage of mathematical structure requires deep conceptual understanding, which must be developed intentionally. This goes beyond the memorizing of facts, formulas, and procedures, and the practicing of skills over and over again (Sam & Ernest, 2000). Students with robust conceptual and procedural understanding are able to take a step back, examine structure, and, with intention, develop a solution method. Approaching problems in this way can reward students with additional time on the SAT, versus the often more time consuming algebraic methods. (These methods are always an option if students are unable to notice opportunities to leverage structure.)

Teachers should:
- Expose students to representations that can be interpreted in multiple ways (SMP 7)
- Support students as they examine and discuss mathematical structure (SMP 7)
- Encourage students as they examine and discuss connections among tasks and representations in tasks (SMP 7)
- Encourage students to view parts of a symbolic expression in relation to the other parts of that expression (SMP 2, SMP 7)
- Ask students questions that they can efficiently answer by using the structures of mathematical equations, expressions, or formulas (SMP 2, SMP 5, SMP 7)
- Use tasks that feature structure in mathematical representations other than algebraic equations and expressions (e.g., tables, graphs, verbal contexts, and written contexts) (SMP 2, SMP 7)

KEY CONSIDERATION

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Practice 4: Assess students formatively and summatively using a variety of methods and task formats.

For any mathematics assessment, students can demonstrate their understanding when they become familiar with problem formats and vocabulary. They must also develop flexible critical-thinking strategies and problem-solving strategies. This is particularly true for tests like the SAT Suite of Assessments.

Teachers should rely on a variety of tasks and problems that share features with SAT items. These should be integrated into instructional practice, as well as formative and summative assessments. This integration should first happen during instruction, with formative assessments guiding students’ learning. On the occasional summative assessments, a few similar problems may also be included. These items should not be a primary focus of instruction, nor should they form a large portion of any particular assessment. This intentional interweaving, however, ensures students will have varied experiences with these problem types over multiple years. At the same time, this maintains the integrity and coherence of curriculum, instruction, and assessment.

**Teachers should:**
- Integrate SAT-like items into instruction and practice, and connect them to curricular goals
- Integrate items when they match the topic, while embedding other items that prompt students to use all strategies (SMP 1, SMP 5)
- Model and support habits that leverage structure (SMP 7), multiple representations (SMP 4), and conceptual understanding
- Model and support sense-making within varied contexts (SMP 1, SMP 4)
- Provide frequent opportunities (small group, whole class) for students to explain how they saw a given problem, decided on a solution strategy, and carried through to a justifiable solution (SMP 2, SMP 3)
- Value students’ thinking and persistence during instruction and assessment, by using their errors and areas of struggle as opportunities to build knowledge and understanding (SMP 2, SMP 3, SMP 6)
- Use strategic questioning to raise students’ sensitivity to mathematical structure and efficiency (SMP 5, SMP 7, SMP 8)
- Create frequent opportunities for students’ thinking to be made visible during assessment and instruction; uncover and discuss students’ thinking and misconceptions, and provide specific, descriptive, and actionable feedback to students so they can grow and improve their understanding.
References


